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Few-body correlations in the QCD phase diagram

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Abstract. From the viewpoint of statistical physics, nuclear matter is a strongly correlated many-particle system. Several regimes of the QCD phase diagram should exhibit strong correlations. Here I focus on three- and four-body correlations that might be important in the phase diagram.

PACS. 21.65.+f Nuclear matter – 21.45.+v Few-body systems

1 Introduction

Lattice and effective model calculations provide a rich and exciting sketch of the phase diagram of Quantum Chromodynamics (QCD). Two regions may coarsely be distinguished: a hadronic phase and a plasma phase. Since quarks are considered part of the fundamental building blocks of matter, the existence of a hadronic phase is already an indication of strong correlations between quark-antiquark and three-quarks. Further on, in both phases correlations lead to more interesting phenomena, such as clustering of nucleons to form nuclei, or superfluidity (in nuclear matter) and color superconductivity (in quark matter). In particular in the later cases a weak residual interaction is enough to destabilize the ground state (just as is the case for the formation of Cooper pairing). These investigations are usually based on the study of two-particle correlations. There are reasons to go beyond two-particle correlations, *e.g.*:

- Particle production even in a dense environment such as deuteron formation in a heavy-ion reaction, need a third particle to conserve energy momentum [1].
- To study the properties of α-particles [2] or determine the critical temperature of a possible α-particle condensate [3–5] needs an in-medium four-body equation.
- Recent results in the Hubbard model indicate, that three-particle contributions may lead to a different (lower) critical temperature compared to the simple Thouless criterion [6]. Question of this type have not been addressed for nuclear matter.
- The chiral phase transition is often discussed along with a confinement-deconfinement transition based on investigating mesons (quark-antiquark states), see, *e.g.*, ref. [7]. Does this transition happen for nucleons (three-quark states) at the same density/temperature?

To investigate these issues a first step is to develop and solve proper effective in-medium three- and four-body equations that are valid at *finite temperatures and densities* analogous to the Feynman-Galitskii or Bethe-Goldstone equations [8]. This has been achieved in the past for the nonrelativistic problem [2,9–11]. These equations have been derived on the basis of statistical Green functions [8]. The Green functions have been decoupled utilizing a cluster expansion, see, *e.g.*, [12]. To tackle these questions in the (deconfined) quark-phase, in addition, such in-medium few-body equations must obey special relativity. As chiral symmetry breaking is presumably restored (up to small current masses) the quarks may become very light objects and therefore relativistic effects should play a larger role than for isolated systems. Here, relativity is realized using the light front form of relativistic dynamics [13]. First results are given in refs. [14,15] on the confinement-deconfinement (Mott) transition.

2 Theory

We use Dyson equations to tackle the many-particle problem, see, *e.g.*, ref. [12]. This enables us to decouple the hierarchy of Green functions. The Dyson equation approach used here is based on two ingredients: i) all particles of a cluster are taken at the same global time, ii) the ensemble averaging for a cluster is done for an uncorrelated medium. The resulting decoupled Green functions may be economically written as resolvents in the n-body space, where $n = 2, 3, 4, \ldots$ is the number of particles in the considered cluster.

The solution of the one-particle problem in the Hartree-Fock approximation leads to the following quasiparticle energy:

$$\varepsilon_1 = \frac{k_1^2}{2m_1} + \sum_2 V_2(12, \widetilde{12}) f_2 \simeq \frac{k_1^2}{2m_1^{\text{eff}}} + \Sigma^{\text{HF}}(0). \qquad (1)$$

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The last equation introduces the effective mass that is a valid concept for the rather low densities considered here and $\mu^{\text{eff}} \equiv \mu - \Sigma^{\text{HF}}(0)$. The Fermi function $f_i \equiv f(\varepsilon_i)$ for the i-th particle is given by

$$f(\varepsilon_i) = \frac{1}{e^{\beta(\varepsilon_i - \mu)} + 1}\,. \tag{2}$$

The resolvent G_0 for n noninteracting quasiparticles is

$$G_0(z) = (z - H_0)^{-1} N \equiv R_0(z) N, \quad H_0 = \sum_{i=1}^{n} \varepsilon_i, \tag{3}$$

where G_0, H_0, and N are formally matrices in n-particle space. The Matsubara frequency z_λ has been analytically continued into the complex plane, $z_\lambda \to z$ [8]. The Pauli blocking for n-particles is

$$N = \bar{f}_1 \bar{f}_2 \ldots \bar{f}_n \pm f_1 f_2 \ldots f_n, \quad \bar{f} = 1 - f, \tag{4}$$

where the upper sign is for Fermi-type and the lower for Bose-type clusters. The full resolvent $G(z)$ is given by

$$G(z) = (z - H_0 - V)^{-1} N, \quad V \equiv \sum_{\text{pairs } \alpha} N_2^\alpha V_2^\alpha. \tag{5}$$

Note that $V^\dagger \neq V$. For the two-body case as well as for a two-body subsystem embedded in the n-body cluster the standard definition of the t-matrix leads to the Feynman-Galitskii equation for finite temperature and densities [8],

$$T_2^\alpha(z) = V_2^\alpha + V_2^\alpha N_2^\alpha R_0(z) T_2^\alpha(z). \tag{6}$$

Introducing the Alt-Grassberger-Sandhas (AGS) [16] transition operator $U_{\alpha\beta}(z)$ the effective inhomogeneous in-medium AGS equation reads

$$U_{\alpha\beta}(z) = \left(1 - \delta_{\alpha\beta}\right) R_0^{-1}(z) + \sum_{\gamma \neq \alpha} N_2^\gamma T_2^\gamma(z) R_0(z) U_{\gamma\beta}(z). \tag{7}$$

The homogeneous in-medium AGS equation uses the form factors defined by

$$|F_\beta\rangle \equiv \sum_\gamma \bar{\delta}_{\beta\gamma} N_2^\gamma V_2^\gamma |\psi_{B_3}\rangle \tag{8}$$

to calculate the bound state ψ_{B_3}

$$|F_\alpha\rangle = \sum_\beta \bar{\delta}_{\alpha\beta} N_2^\beta T_2^\beta(B_3) R_0(B_3) |F_\beta\rangle. \tag{9}$$

Finally, the four-body bound state is described by

$$\begin{aligned} \left|\mathcal{F}_\beta^\sigma\right\rangle = \sum_{\tau\gamma} &\bar{\delta}_{\sigma\tau} U_{\beta\gamma}^\tau(B_4) R_0(B_4) \\ &\times N_2^\gamma T_2^\gamma(B_4) R_0(B_4) \left|\mathcal{F}_\gamma^\tau\right\rangle, \end{aligned} \tag{10}$$

where $\alpha \subset \sigma$, $\gamma \subset \tau$ and σ, τ denote the four-body partitions. The two-body input is given in (6) and the three-body input by (7). Note that, although we have managed to rewrite the above equations in a way close to the ones for the isolated case, they contain all the relevant in-medium corrections in a systematic way, *i.e.* correct Pauli-blocking and self-energy corrections. The numerical solution requires some mild approximations that are however well understood in the context of the isolated few-body problem.

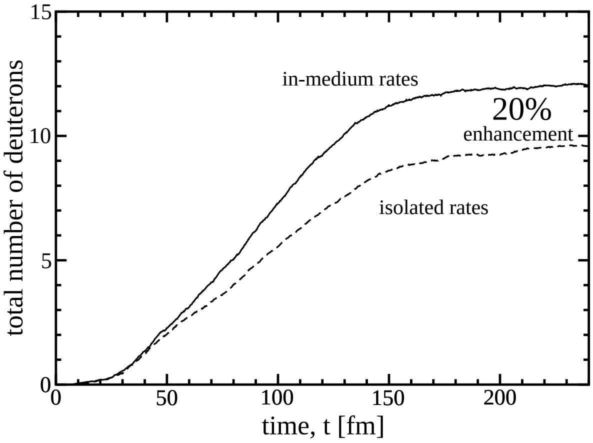

Fig. 1. BUU simulation of the deuteron formation during the central collision of ^{129}Xe + ^{119}Sn at 50 MeV/A.

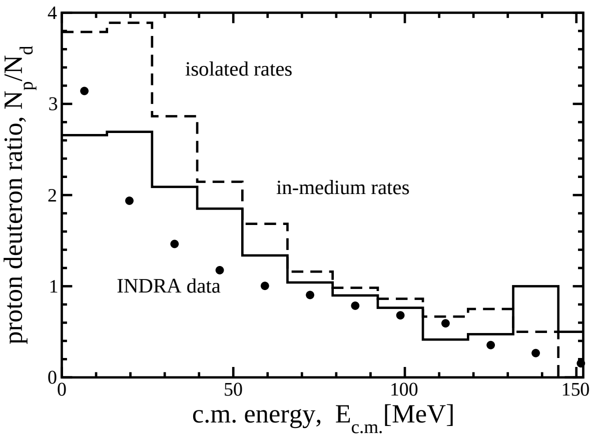

Fig. 2. Ratio of proton to deuteron numbers as a function of c.m. energy. The experimental data are from the INDRA Collaboration.

3 Results

An experiment to explore the equation of state of nuclear matter is heavy-ion collisions at various energies. Here we focus on intermediate to low scattering energies and compare results to a recent experiment ^{129}Xe + ^{119}Sn at 50 MeV/A by the INDRA Collaboration [17]. A microscopic approach to tackle the heavy-ion collision is given by the Boltzmann equation for different particle distributions and solved via a Boltzmann-Uehling-Uhlenbeck (BUU) simulation [18,19]. The reaction rates appearing in the collision integrals are *a priori* medium dependent. However, previously this medium dependence has been neglected. Within linear response theory for infinite nuclear matter the use of in-medium rates leads to faster time scales for the deuteron lifetime and the chemical relaxation time, as has been shown in detail in refs. [20,21]. This faster time scales should have consequences for the freeze-out of fragments.

We use the in-medium AGS equations (7) that reproduce the experimental data in the limit of an isolated three-body system. For details on the specific interaction

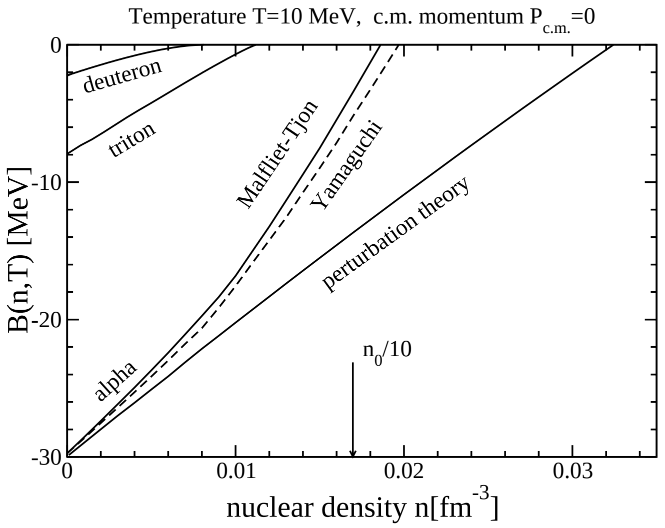

Fig. 3. Difference between the pole energy of the bound state and the continuum, $B(n,T) = E_{\rm pole} - E_{\rm cont}$.

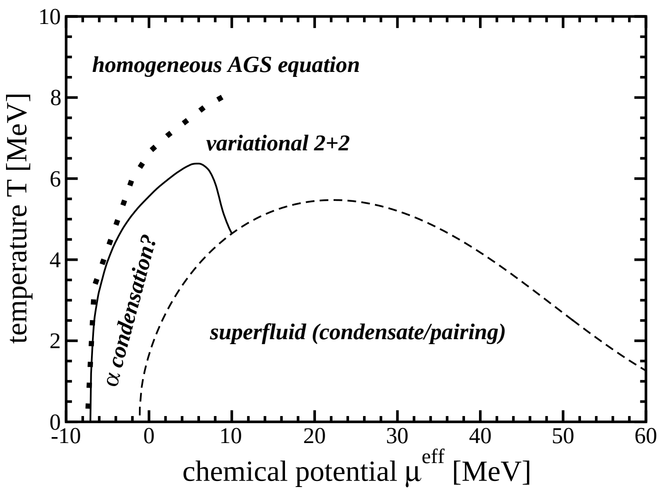

Fig. 4. Critical temperatures of condensation/pairing leading to superfluid nuclear matter. For an explanation see text.

model see ref. [9]. We investigate the influence of medium-dependent rates in the BUU simulation of the heavy-ion collision as compared to the use of isolated (*i.e.* experimental) rates. Figure 1 shows that the net effect (gain-minus-loss) of deuteron production becomes larger for the use of in-medium rates (solid line) compared to using the isolated rates (dashed line). The change is significant, however, a comparison with experimental data is difficult since deuterons may also be evaporating from larger clusters that have not been taken into account in the present calculation so far. The ratio of protons to deuterons may be better suited for a comparison to experiments that is shown in fig. 2. The use of in-medium rates (solid line) lead to a shape closer to the experimental data (dots) than the use of isolated rates (dashed line). In these calculation, besides the change of rates, also the Mott effect has been taken into account.

Figure 3 shows the dependence of the binding energy for different clusters at a given temperature $T = 10\,\mathrm{MeV}$ and at rest in the medium.

In fig. 4 part of the phase diagram of nuclear matter is shown. The condition for the onset of superfluidity for α-particles is $B(T_c, \mu, P = 0) = 4\mu$. The critical temperature found by solving the homogeneous AGS equation for $\mu < 0$ confirms the onset of α condensation even at higher values (dotted line) than given earlier (solid line, from [3]) based on a variational calculation using the $2 + 2$ component of the α-particle. For $\mu > 0$ the condition $E = 4\mu$ for the phase transition can also be fulfilled. However, the significance for a possible quartetting needs further investigation. Due to the many-channel situation of more than two-particles equations, the Thouless criterion might be revisited.

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